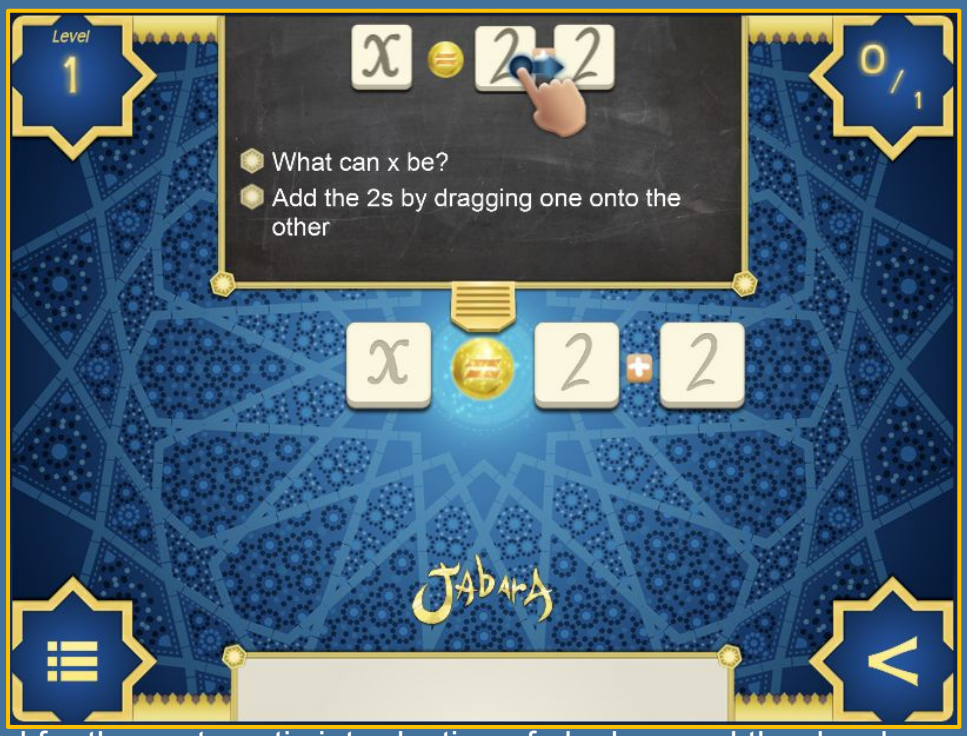
Paper Transfer Kit

Mangahigh.com

What is Jabara ?

Jabara is an algebra game that your students can use on desktops, iPads, and smartphones. Jabara teaches algebra concepts, but also introduces vocabulary and enrichment.



Designed for the systematic introduction of algebra, and the development of robust simplification skills, Jabara offers:

- 180+ beautiful and cleverly designed levels
- Satisfying and addictive gameplay
- Modern pedagogy and concise explanations alongside gradual difficulty progression
- Social competition and Achievements
- Enrichment and maths history
- (On Mangahigh.com) Easy and powerful task setting and full analytics

You can play Jabara in the browser on iPads, Laptops or phones!

Get the game on Mangahigh.com!

This Paper Transfer Kit is designed to consolidate your students' games-based algebra skills, and to help them to reproduce those skills on paper

Modern maths games like Jabara get your students thinking about hard algebra concepts and practicing, but transferring digital mathematics skills to paper remains an important priority. Your students will not fully benefit unless you complement their games-based learning with paper work.

The Jabara Paper Transfer Kit is designed to reinforce digital skills learned in Jabara, and also to assess student progress. Each exercise has brief student instructions on how it should be completed, and is designed to follow on from playing Jabara. The worksheet uses terms introduced during the game, and asks students to replicate the work they have done digitally with pen and paper.

Each set of Exercises in the Paper Transfer Kit are associated with one of the game Challenges. The Game Challenges are groups of 20 levels, each Challenge being represented by different-coloured segments in the Level Select wheel, and a specific Achievement (sweetie, lollipop etc.). You should ask your students to complete exercises from the Paper Transfer Kit as they complete the Challenges in the game for maximum benefit.

Get the game on Mangahigh.com!

JABARA
Transferring Digital Skills to Paper

Game Level	Aim of Challenge	Paper Transfer Exercise
L1 - L20	**1** *Simplifying constants*	**1.1** Variables and Constants
		1.2 Simplifying Constants
		1.3 Constants & Zeroes
		1.4 Isolating the Variable
		1.5 Working with Negatives
		1.6 Word Problems
L21 - L40	**2** *Adding to both sides*	**2.1** Adding to Both Sides
		2.2 Balancing Equations
		2.3 Word Problems
L41 - L60	**3** *Coefficients, like terms, multiplying constants*	**3.1** True or False
		3.2 Simplifying Like Terms
		3.3 Simplifying (Linking)
		3.4 Simplifying and Showing Workings
		3.5 Word Problems
L61 - L80	**4** *Unknowns on both sides, adding variables, swapping sides*	**4.1** Adding variables to both sides
		4.2 Spotting Mistakes
		4.3 Rearranging Equations
		4.4 Word Problems

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JABARA
Transferring Digital Skills to Paper

Game Level	Aim of Challenge	Paper Transfer Exercise
L81 - L100	**5** *Fractions, dividing both sides*	**5.1** True or False
		5.2 Code Breaker
		5.3 Word Problems
L101 - L120	**6** *Multiplying both sides, adding fractions with like denominators*	**6.1** Simplifying Equations (Linking)
		6.2 Solving Equations and Showing Workings
		6.3 Spotting Mistakes
		6.4 Word Problems
L121 - L140	**7** *Brackets 1 - Brackets, and bracketed terms multiplied by constants*	**7.1** Expanding Brackets
		7.2 Solving Equations and Showing Workings
		7.3 Word Problems
L141 - L160	**8** *Factors, adding fractions with unlike denominators*	**8.1** Combining Fractions
		8.2 True or False
		8.3 Solving Equations and Showing Workings
		8.4 Word Problems

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Challenge 1: Simplifying Constants

1.1 Variables and Constants

A ***variable*** is a letter, like x or y

A ***constant*** is a number on its own, like 5

Draw a **CIRCLE** around the ***constants*** (numbers)

Draw a **SQUARE** around the ***variables*** (letters)

The first two are done for you.

1	x	2	p	17
y	6	8	100	t

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1.2 Simplifying Constants

When you're solving equations always
simplify the constants
by *adding* or *subtracting* the numbers.

Tidy up these equations by ***simplifying the constants***.

The first one is done for you.

Equation		Simplified
$x = 2 + 4$	⇒	$x = 6$
$x = 3 + 5$	⇒	
$x = 7 - 3$	⇒	
$x = 2 + 3 + 1$	⇒	
$y = 12 + 5$	⇒	
$y = 5 + 2 - 3$	⇒	
$y = 10 - 3 + 2$	⇒	

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Challenge 1: Simplifying Constants

1.3 Constants & Zeroes

When you're solving equations you sometimes need to tidy them up by:

removing zeros

AND

simplifying the constants.

Tidy up these equations.

The first one is done for you.

Equation		Tidied up
$x + 0 = 1 + 6$	→	$x = 7$
$x = 2 + 7 + 0$	→	
$0 + x = 4 + 2$	→	
$x + 0 = 9 - 5$	→	
$0 + y = 8 - 5$	→	
$y + 0 = 5 + 2 + 3$	→	
$0 + y = 9 + 0 - 3$	→	

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1.4 Isolating the Variable

When you ***isolate the variable*** on one side of the equals sign you can then solve the equation.

Think of it as ***'getting the letter on its own***'

Here the variable is isolated: $x = 2 + 9$

Here the variable is **NOT** isolated: $x + 5 = 7$

Put a circle around each equation that already has the ***variable isolated*** (alone). The first one is done for you.

$$x = 2 + 9$$

$$x = 5 + 7 + 3$$

$$y + 6 = 1$$

$$y = 0 + 2 + 9$$

$$x + 4 = 8 + 6$$

$$x = 7 - 4$$

$$7 + 7 = x - 3$$

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Challenge 1: Simplifying Constants

1.5 Working with Negatives

Fill in the boxes to solve each equation.

Question 1

$x = 5 + (-3)$

$x =$ ☐

Look!
Adding (−3) is the same as subtracting 3

Question 2

$x = (-1) + 4$

$x =$ ☐

Question 3

$x = 5 + 2 + (-2)$

$x = 5 +$ ☐

$x =$ ☐

Question 4

$x = 7 + 4 + (-3)$

$x = 7 +$ ☐

$x =$ ☐

Question 5

$x = 6 + (-2) + 4 - 4$

$x =$ ☐ + ☐

$x =$ ☐

Question 6

$x = 3 + (-4) + (-7) + 7$

$x =$ ☐ + ☐

$x =$ ☐

Question 7

$x = (-3) + 7 + 5 + (-5)$

$x =$ ☐ + ☐

$x =$ ☐

Question 8

$x = (-2) + 2 + 3 + (-3)$

$x =$ ☐ + ☐

$x =$ ☐

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1.6 Word Problems

Example

A camel walks 5 miles due East, then another 4 miles East.
She then retraces her steps for 3 miles.
How far from her starting point is she?

Let x be the distance from the starting point.

$x = 5 + 4 - 3$ (*form the equation*)
$x = 6$ (*simplify constants*)

So the camel is 6 miles East from her starting point.

Now it's your turn!

Darius has 4 gold pieces.
A thief steals 2 of his gold pieces, but he then earns 5 more. How many gold pieces does Darius now have?

Let x equal the number of gold pieces he finishes with.

SHOW YOUR WORKINGS!

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2.1 Adding to Both Sides

Lots of equations can be solved by
adding a constant to both sides of the equation.
Think of equations like scales.
Add the same amount to each side so that they balance.

Put a circle around the constant that you need to
add to both sides of the equation to help solve it.

Question 1 is done for you.

Question 1

x + -3 = 7

6 3 x 2

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Challenge 2: Adding to Both Sides

Question 2

$$x + 5 = 8 + 3$$

-5 | 8 | 5 | 3

Question 3

$$x + -1 = 6 + 5$$

6 | 0 | -1 | 1

Challenge 2: Adding to Both Sides

Question 4

$x + 2 = 3 + 2$

3 | -x | -2 | -3

Question 5

$x + 1 + 8 = 6 + 5$

9 | -9 | -8 | 8

Challenge 2: Adding to Both Sides

2.2 Balancing Equations

It is important to show all your working when you solve an equation - even when you can '*see*' the answer!

Question 1

$$x + 4 = 9$$

$$x + 4 + \square = 9 + \square$$

$$x = \square$$

Add to both sides..

HINT:
Use a number from this box

4	−4	9	−9
0	−5	5	1

Question 2

$$x - 2 + 3 = 5 + 4$$

Simplify constants

$$x + \square = \square$$

Add to both sides..

$$x + \square + \square = \square + \square$$

$$x + \square = \square$$

$$x = \square$$

HINT:
Use a number from this box

1	−1	9	5
0	−5	8	10

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2.2 Balancing Equations (contd.)

The variable doesn't have to be x...other letter symbols can be used!

Fill in the boxes to complete each solution.

Question 3

$y - 6 = 9 - 4$

$y - 6 =$ ☐

$y -$ ☐ ☐ ☐

Add to both sides

$y =$ ☐

HINT:
Use a number from this box

6	−6	9	−9
11	−11	1	−1

Question 4 Now it's your turn….

$y - 2 + 1 = 5 - 2$

HINT:
Use a number from this box

1	−1	3	2
−2	−3	4	0

Question 5

$4 + z - 2 = 1 + 3 - 5$

HINT:
Use a number from this box

1	−2	2	3
4	−1	0	−3

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Challenge 2: Adding to Both Sides

2.3 Word Problem

Example

A pomegranate and a jar of olives cost 12 bronze coins
A jar of olives costs 4 bronze coins.
How much does a pomegranate cost?

Let p represent the cost of one pomegranate.

$$p + 4 = 12 \quad \text{(}\textit{Form the equation}\text{)}$$
$$p + 4 + (-4) = 12 + (-4) \quad \text{(}\textit{Add to both sides to isolate } p\text{)}$$
$$p + 0 = 8 \quad \text{(}\textit{Simplify constants}\text{)}$$
$$p = 8 \quad \text{(}\textit{Remove zeros}\text{)}$$

So a pomegranate costs 8 bronze coins.

Now it's your turn!

Jila has some eggs.
She breaks 4 eggs and buys six more.
At the end she has 18 eggs.
How many did she start with?

Let x represent the number of eggs Jila starts with.

$x + (\ \square\ ) + \square = 18$ (*Form the equation...*)

(...*now solve it!*)

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3.1 True or False

Tick the statements that are always true.

Correct any statements that are false.

$1x = x$ ☐ _____

$0x = 1$ ☐ _____

$+3 + -3 = 0$ ☐ _____

$(-4x) + (4x) = 0$ ☐ _____

$-2 + -2 = 0$ ☐ _____

$5x + (-5x) = 0$ ☐ _____

$4x + (-5x) = x$ ☐ _____

$-4 + x + 4 = x$ ☐ _____

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3.2 Simplifying Like Terms

Simplify each expression in the left hand column of the table.
The first one is done for you!

Expression	*Simplified expression*
$5 \times (-3)$	-15
$4 + 2 \times 3$	
$1x$	
$-x + 2x$	
$0x$	
$-1x + x$	
$x + x + x$	
$x + 2x - x$	
$-2x + 3 + 2x$	
$-3 + 2x + 3$	
$-x + 2x + 3 - 3$	
$6 - 3x - 3x - 2$	

Challenge 3: Coefficients, Like Terms, Multiplying Constants

3.3 Simplifying (Linking)

Alina has simplified each of these equations so she can solve them.
Match together the equations, simplified equations and solutions.

Equation	Simplified Equation	Solution
$3\times2\times x+(-5)\times x=4+3$	$3x-4x+2x=-2$	$x=14$
$3\times3\times x-4\times2\times x=4+3\times2$	$6x-5x=7$	$3=x$
$2\times(-7)\times x+5\times3\times x=2\times7$	$2x-2x+3=-3x+4x$	$x=7$
$3\times x+(-4)\times x+2\times x=2-4$	$-14x+15x=14$	$x=-2$
$1\times x+(-1)\times x+1=(-4)\times x+5\times x$	$x-x+1=-4x+5x$	$x=10$
$2\times x+2+(-2)\times x+1=(-3)\times x+4\times x$	$9x-8x=10$	$1=x$

This one has been done for you!

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Challenge 3: Coefficients, Like Terms, Multiplying Constants

3.4 Simplifying and Showing Workings

Complete the missing working to show how you solve each equation.
You must always show each stage of your solution on a new line.

Question 1

$3x + 4x + 3 - 6x = 2x + 7 - 2x$

$x + 3 = 7$ *(simplify)*

$x + 3 + (-3) = 5 + (-3)$ *(isolate x)*

$x = 4$ *(simplify and solve!)*

This one is done for you...

Question 2

$5x + 4 - 4x = 3 \times 4$

_____ *(simplify)*

_____ *(isolate x)*

_____ *(simplify and solve!)*

...now you do the rest!

Question 3

$6x - 2x - 2 - 3x = 2 \times 4 - 5$

_____ *(simplify)*

_____ *(isolate x)*

_____ *(simplify and solve!)*

Remember: The variable can be any letter!

Challenge 3: Coefficients, Like Terms, Multiplying Constants

Question 4

$7x + 5 - 2 \times 3x = 3 - 2$

$x + 5 = 1$ _____

$x + 5 + (-5) = 1 + (-5)$ _____

$\underline{x = -4}$ _____

Explain what happens at each stage in the solution.

Question 5

$5 \times 2y - 4 - 3 \times 3y = 7 - 2 \times 3$

You're on your own for these last two…

Question 6

$z + z + 3 \times 2 - 2z = 5z - 2 \times 2z + 3$

...don't forget to show all your working!

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3.5 Word Problem

EXAMPLE

Darius buys 2 bags of 4 melons and 3 bottles of oil.
On the way home, Darius sells 7 of his melons to Alina
for the same price that he paid for them.
Altogether, Darius has spent 13 gold coins.

One bottle of oil costs 4 gold coins.
Find the cost of one melon.

Let m be the cost of one melon.

$8m + 3 \times 4 - 7m = 13$ (*form the equation*)

Now you solve the equation!

Now it's your turn!

Alina buys 3 bags of 5 figs and 3 loaves of bread.
On the way home, Alina sells 14 of her figs to Jameela
for the same price that she paid for them.
Altogether, Alina has spent 20 bronze coins.

One loaf of bread costs 6 bronze coins.
Find the cost of one fig.

SHOW YOUR WORKINGS.

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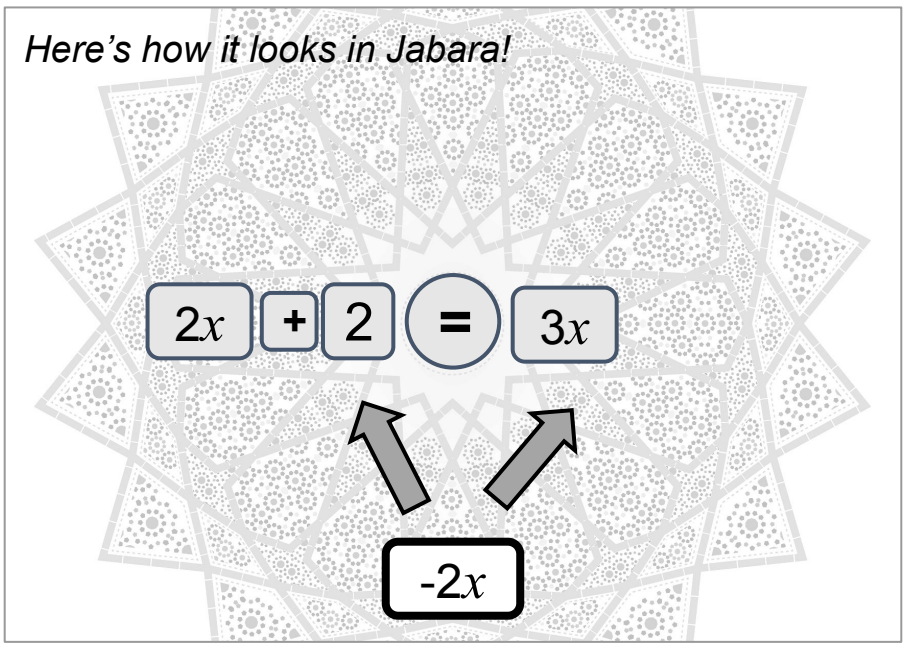
4.1 Adding variables to both sides

Sometimes x is on both sides of the equation!

You need to eliminate x from one side by adding negative x to BOTH sides to keep the equation balanced

Choose the right terms from the tray to get rid of the x on one side of each equation. You can use each term more than once!

Here's how it looks in Jabara!



Now it's your turn!

$3x + 3 = 2x + 1$

$3x + 3 +$ $= 2x + 1 +$

$=$ *(Simplify each side)*

(Complete the solution)

Hint

$2x$ $-2x$

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4.2 Spotting Mistakes

Alina's Algebra master has set her some equations to solve.
Here are Alina's solutions - but they are all wrong!
Find and correct all of Alina's mistakes.

$$3x + 2x + 3 = 4x - 3$$
$$5x + 3 = 4x - 3$$
$$5x - 4x + 3 = -3$$
$$x + 3 = -3$$
$$x = -3 + 3$$
$$\underline{x = 6}$$

$$x - 2 + x = 3x - 5$$
$$-2 = 3x - 5$$
$$-2 + 5 = 3x - 5 + 5$$
$$3 = 3x$$
$$\underline{1 = x}$$

$$-3x - x + 2 = 6 - 2x - x$$
$$-4x + 2 = 6 - 3x$$
$$x + 2 = 6$$
$$x = 6 - 2$$
$$\underline{x = 4}$$

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4.3 Rearranging Equations

Use the maths hack to help you solve these equations.

$4x + 5x = 4x + 4x + 4$

_____ (*Simplify*)

_____ (*Put all the x's on one side*)

_____ (*Simplify and solve!)*

$3x + 2x + x - 3 = 5x + 7$

_____ (*Simplify*)

_____ (*Put all the x's on one side...*)

_____ (*...and the constants on the other)*

_____ (*Simplify and solve!*)

$-6y + 3 = 3y + 2y - 10y + 4$

_____ (*Simplify*)

_____ (*Put all the y's on one side...*)

_____ (*...and the constants on the other)*

_____ (*Simplify and solve!*)

$7z - 5z - 2 = 5z - 2z - 6$

Try this one on your own!

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4.4 Word Problems

Example

Darius and Alina go shopping at the market,
they both spend the same amount of money.
Darius buys 3 camels and 4 goats.
Alina buys 2 camels and 6 goats.

One goat costs 20 gold coins.
Find the cost of one camel.

Let c be the cost of one camel

$3c + 4 \times 20 = 2c + 6 \times 20$ (*form the equation*)

Now solve the equation!

Now it's your turn!

Darius and Alina go shopping at the market,
they both spend the same amount of money.
Darius buys 4 cows and 7 sheep.
Alina buys 5 cows and 3 sheep.

One sheep costs 15 gold coins.
Find the cost of one cow.

Let c be the cost of one cow.

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Challenge 5: Fractions, Dividing Both Sides

5.1 True or False

You have found some scrolls at the House of Wisdom, but some of them have been altered by Monge Khan.

Tick the statements on the scroll which are ***always true***.
Put a cross by, and correct any statements that are false.

$\frac{15x}{15} = x$ ☐

$\frac{3x}{3} = 1$ ☐

$\frac{12x}{6} = 2x$ ☐

$\frac{10x}{10} = 0$ ☐

$\frac{3y}{6} = 2y$ ☐

$\frac{2 \times 4 \times 6x}{3 \times 4} = 3x$ ☐

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5.2 Code Breaker

The masters at the House of Wisdom were very secretive.
They used algebra to send coded messages.
Solve these equations in order to crack the code.

1. $3x = 9$

2. $\frac{3 \times 4x}{12} - 2 = 4$

3. $\frac{6x}{2 \times 3} + 2x = 1 + 2 + 3$

4. $6x = 6$

5. $\frac{20x}{2 \times 5} + 5 = 2 + 2 \times 2$

6. $18 = 10 - 4x$

3 = A	$\frac{1}{2}$ = B	-3 = C	1 = E	2 = G	0 = H
$\frac{1}{3}$ = K	6 = L	5 = N	-2 = R	-1 = S	4 = T

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Challenge 5.2 Code Breaker cont.

7. $2x - 3 = 3$

8. $\frac{4x}{4} + 3 = 10 - 7$

9. $5x - 3 = 3x + 3$

10. $x - 6 = x + x + x$

11. $12x - 3 = 1$

12. $2x - 4 = 5x - 7$

13. $x + 2x + 3x = 8x + 4$

3 = A	$\frac{1}{2}$ = B
-3 = C	1 = E
2 = G	0 = H

$\frac{1}{3}$ = K	6 = L
5 = N	-2 = R
-1 = S	4 = T

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5.3 Word Problems

Can you solve a problem that is more than 1000 years old?

A man is hired to work in a vineyard
for 30 days for 10 gold coins.
He works 6 days.
How much should he receive?

Let x be the number of gold coins the man receives.

SHOW YOUR WORKINGS!

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6.1 Simplifying Equations (Linking)

Darius is solving some equations.

Here are his starting equations...

...and here is part of each solution.

Match together the equivalent equations.

The first one is done for you!

$\frac{5x}{5} = 2$	$x + 3 = 2$
$\frac{15x}{5} = 9$	$x = 2$
$\frac{x}{7} + 3 = \frac{2}{7} + 5$	$5 = x + 3$
$\frac{10 \times 2}{3} = x + 1$	$3x = 9$
$x + 3 = \frac{2}{5} + \frac{3}{5} + \frac{5}{5}$	$x + 21 = 2 + 35$
$5 = \frac{5x + 15}{5}$	$20 = 3x + 3$

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6.2 Solving Equations and Showing Workings

Mongke Khan has vandalized these scrolls.

Can you fill in the gaps?

Question 1

$$\frac{4x+8}{8} + \frac{x}{2} = 5$$

$$\frac{4x}{8} + \square + \frac{x}{2} = 5$$ *(Split the fraction)*

$$\frac{x}{\square} + \frac{x}{2} = \square$$ *(Simplify and move the constant to the other side)*

Now finish off the solution!

Question 2

$$\frac{12x-9}{3} = x + 3$$

$$\frac{12x}{3} - \square = x + 3$$ *(Split the fraction)*

Now finish off the solution!

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Question 3

$$\frac{6-2x}{2} = \frac{5x+10}{5}$$

$\square - \square = \square + \square$ *(Split the fractions)*

Now finish off the solution!

Question 4

Try this one on your own!

$$\frac{x}{60} + \frac{x}{20} = \frac{1}{3}$$

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6.3 Spotting Mistakes

Darius is training to become an algebra apprentice, but he keeps going wrong. Can you help him out? For each equation: find Darius's mistake, correct it and solve the equation.

$$3 = \frac{2x + 4}{2}$$

$$3 = x + 4$$

$$3 - 4 = x$$

$$-1 = x$$

$$x = \frac{x}{2} - 4$$

$$x - \frac{x}{2} = 4$$

$$2x - x = 8$$

$$x = 8$$

$$\frac{2x}{5} - \frac{x}{3} = 1$$

$$2x - \frac{x}{3} = 5$$

$$6x - x = 15$$

$$5x = 15$$

$$x = 3$$

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6.4 Word Problems

Example

Alina travels by camel to the market at 4 leagues per hour to sell some silks.
On her way home, she travels at 5 leagues per hour.
Her total journey time is 9 hours.

How far away is the market from Alina's house?

Let d = the distance from Alina's house to the market

$\frac{d}{4} + \frac{d}{5} = 9$ (*form the equation*)

Hint: *distance = time / speed*

Now you solve the equation!

Now it's your turn!

Darius goes for a run.
He leaves home and runs at 3 leagues per hour to the oasis.

On his way home, he runs at 2 leagues per hour.
His total journey time is 1 hour.

How far away is the oasis from Darius's house?

Let d = the distance from Darius's house to the oasis.

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7.1 Expanding Brackets

Alina has expanded some expressions,
but her answers are all muddled up.
Can you sort them out?

Here are her starting expressions...

...and here are Alina's answers

Match together the equivalent expressions.

$6(2+4)$	$6x$
$2(2x+x)$	$6-x$
$3(4+x)$	36
$6(2-x)$	$3x-6$
$3(x-2)$	$12-6x$
$2(3+x)-3x$	$12+3x$

The first one is done for you!

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7.2 Solving Equations and Showing Workings

Complete the working to solve each equation.

Question 1

$8 = 3(x + 1) + 2$

_____ (*Multiply out the brackets*)

_____ (*Simplify*)

_____ (*Move the constants onto one side*)

_____ (*And solve!*)

Question 2

$2(x - 4) + 3(x - 1) = 9$

_____ (*Multiply out the brackets*)

_____ (*Simplify*)

_____ (*Move the constants onto one side*)

_____ (*And solve!*)

Question 3

$3(x + 1) - 2(4 - x) = 10$

_____ (*Multiply out the brackets*)

_____ (*Simplify*)

_____ (*Move the constants onto one side*)

_____ (*And solve!*)

7.3 Word Problems

You'll need to use brackets to solve these word problems!

Darius, Alina and Jila share a bag of 185 gold coins.

Alina has 5 less coins than Darius.
Jila has twice as many coins Alina.

How many coins do they have each?

Let n = the number of coins that Darius has

Form an equation first…

...and then solve it!

Darius is 6 years older than Alina.

6 years ago, Darius was twice Alina's age.

How old are Darius and Alina now?

Let d = Darius's age

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Challenge 8: Fractions, Dividing Both Sides

8.1 Combining Fractions

Write each of the following as a single fraction.

Pair of fractions...	Single fraction...
$\frac{1}{2} + \frac{1}{4}$	
$\frac{1}{3} - \frac{1}{6}$	
$\frac{x}{3} + \frac{x}{3}$	
$\frac{3x}{5} + \frac{x}{5}$	
$\frac{9x}{7} - \frac{3x}{7}$	
$\frac{2x}{10} + \frac{x}{5}$	
$\frac{6x}{12} - \frac{x}{4}$	

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8.2 True or False

You have found some scrolls at the House of Wisdom, but some of them have been altered by Monge Khan.

Tick the statements on the scroll which are ***always true***.
Correct any statements which are false.

$\frac{2x}{4} + \frac{x}{2} = x$ ☐

$\frac{5x + 10}{5} = x + 10$ ☐

$\frac{-4y - 2y}{2} = y$ ☐

$\frac{15x}{10} - \frac{x}{2} = x$ ☐

$\frac{14y + 7}{7} = 1 + 2y$ ☐

$\frac{z}{2} - \frac{z}{3} = \frac{z}{5}$ ☐

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Challenge 8: Fractions, Dividing Both Sides

8.3 Solving Equations and Showing Workings

Complete the working to solve each equation.

Question 1

$$\frac{3x}{2} + \frac{4x}{8} = 10$$

_____ (*Simplify the 2nd fraction*)

_____ (*Add the fractions together*)

_____ (*Simplify and solve!*)

Question 2

$$\frac{1}{2} + \frac{8x + 4}{8} = 4$$

_____ (*Split the 2nd fraction*)

_____ (*Simplify*)

_____ (*Move the constants onto one side*)

_____ (*And solve!*)

Question 3

$$\frac{5}{3} + \frac{6x + 5 - x}{15} = 1$$

_____ (*Simplify and split the 2nd fraction*)

_____ (*Simplify*)

_____ (*Move the constants onto one side*)

_____ (*And solve!*)

8.4 Word Problems

Diophantus was known as the Father of Algebra.
Can you solve this ancient riddle to work out his age when he died?

Here lies Diophantus,' the wonder behold.
Through art algebraic, the stone tells how old:

His boyhood lasted one-sixth of his life;
his beard grew after one-twelfth more;
he married after one-seventh more;
and his son was born five years later;
the son lived to half his father's final age,
and Diophantus died four years after his son.

Let d = Diophantus's final age

$$d = \frac{d}{6} + \square + \square + \square + \square + 4$$

Complete the equation….

….remove the fractions…

….then solve the equation.

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Challenge 9: Brackets 2

9.1 Spotting Mistakes

Jila is training to become an algebra apprentice,
but she keeps going wrong.
Can you help her out?
For each equation: find Jilas mistake,
correct it and solve the equation.

$$5(x+2) = 3x - 4$$
$$5x + 2 = 3x - 4$$
$$5x - 3x + 2 = -4$$
$$2x = -4 - 2$$
$$2x = -6$$
$$x = -3$$

$$2(2x+1) = 6x + 6$$
$$4x + 2 = 6x + 6$$
$$2x + 2 = 6$$
$$2x = 6 - 2$$
$$2x = 4$$
$$x = 2$$

$$4(x+1) - 3(x-1) = 8$$
$$4x + 4 - 3x - 3 = 8$$
$$x + 1 = 8$$
$$x = 7$$

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Challenge 9: Brackets 2

9.2 Code Breaker

The masters at the House of Wisdom were very secretive.
They used algebra to send coded messages.
Solve these equations in order to crack the code.

1. $x(7 + (-2)) = 10$

2. $5(3x + 2) = 5 - 10$

3. $2(x - 3) = x$

4. $2(x + x + x) = 3$

5. $4(2x + 4) = 7(3 + x)$

6. $\frac{x}{2} = 3(x - 5)$

-4 = A	= D	5 = E	0 = F	= I	2 = M
= N	4 = O	3 = R	6 = S	$\frac{1}{2}$ = T	= W

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7. $5(x-2)+3(2-x)=x$

8. $2\left(\frac{2x}{4}-3\right)=x-6$

9.

10.

11.

12.

13.

14.

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Challenge 9: Brackets 2

9.3 Word Problems

You'll need to use brackets to solve these word problems!

Let n = the number of coins that Darius has

Form an equation first…

...and then solve it!

Let d = Darius's age

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Answers

Challenge 1: Simplifying Constants

1.1 Variables and Constants

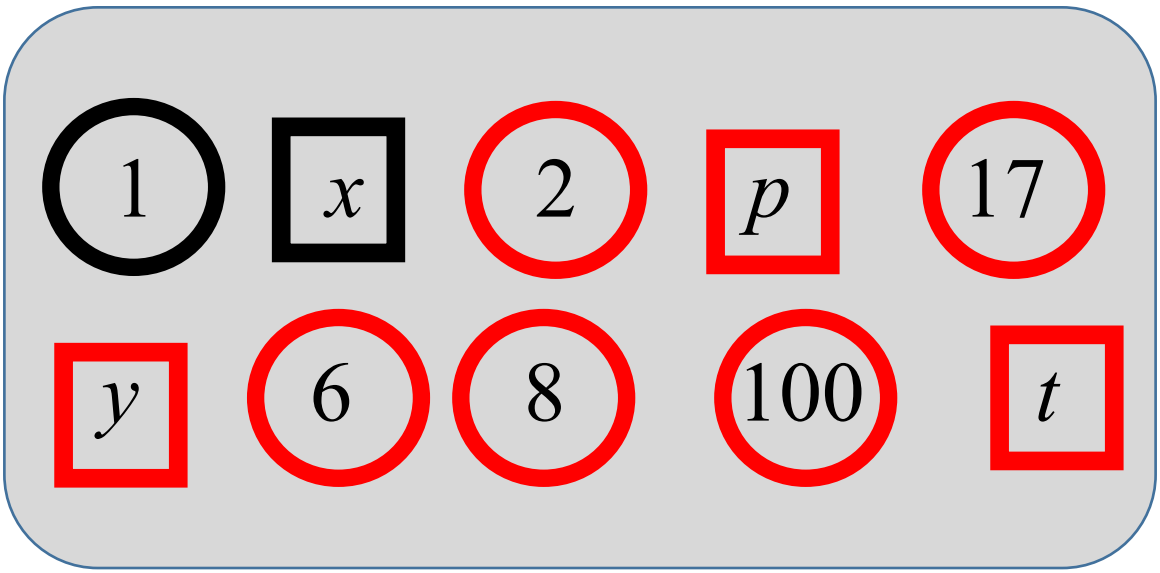
A ***variable*** is a letter, like x or y

A ***constant*** is a number on its own, like 5

Draw a **CIRCLE** around the ***constants*** (numbers)

Draw a **SQUARE** around the ***variables*** (letters)

The first two are done for you.



1.2 Simplifying Constants

When you're solving equations always
simplify the constants
by *adding* or *subtracting* the numbers.

Tidy up these equations by ***simplifying the constants***.

The first one is done for you.

Equation		Simplified
$x = 2 + 4$	⇒	$x = 6$
$x = 3 + 5$	⇒	$x = 8$
$x = 7 - 3$	⇒	$x = 4$
$x = 2 + 3 + 1$	⇒	$x = 6$
$y = 12 + 5$	⇒	$x = 17$
$y = 5 + 2 - 3$	⇒	$x = 4$
$y = 10 - 3 + 2$	⇒	$x = 9$

Challenge 1: Simplifying Constants

1.3 Constants & Zeroes

When you're solving equations you sometimes need to tidy them up by:

removing zeros

AND

simplifying the constants.

Tidy up these equations.

The first one is done for you.

Equation		Tidied up
$x + 0 = 1 + 6$	→	$x = 7$
$x = 2 + 7 + 0$	→	$x = 9$
$0 + x = 4 + 2$	→	$x = 6$
$x + 0 = 9 - 5$	→	$x = 4$
$0 + y = 8 - 5$	→	$y = 3$
$y + 0 = 5 + 2 + 3$	→	$y = 10$
$0 + y = 9 + 0 - 3$	→	$y = 6$

1.4 Isolating the Variable

When you ***isolate the variable*** on one side of the equals sign you can then solve the equation.

Think of it as ***'getting the letter on its own'***

Here the variable is isolated: $x = 2 + 9$

Here the variable is **NOT** isolated: $x + 5 = 7$

Put a circle around each equation that already has the ***variable isolated*** (alone). The first one is done for you.

$$x = 2 + 9$$

$$x = 5 + 7 + 3$$

$$y + 6 = 1$$

$$y = 0 + 2 + 9$$

$$x + 4 = 8 + 6$$

$$x = 7 - 4$$

$$7 + 7 = x - 3$$

Challenge 1: Simplifying Constants

1.5 Working with Negatives

Fill in the boxes to solve each equation.

Question 1

$x = 5 + (-3)$

$x =$ 2

Look! Adding (−3) is the same as subtracting 3

Question 2

$x = (-1) + 4$

$x =$ 3

Question 3

$x = 5 + 2 + (-2)$

$x = 5 +$ 0

$x =$ 5

Question 4

$x = 7 + 4 + (-3)$

$x = 7 +$ 1

$x =$ 8

Question 5

$x = 6 + (-2) + 4 - 4$

$x =$ 4 + 0

$x =$ 4

Question 6

$x = 3 + (-4) + (-7) + 7$

$x =$ −1 + 0

$x =$ −1

Question 7

$x = (-3) + 7 + 5 + (-5)$

$x =$ 4 + 0

$x =$ 4

Question 8

$x = (-2) + 2 + 3 + (-3)$

$x =$ 0 + 0

$x =$ 0

1.6 Word Problems

EXAMPLE

A camel walks 5 miles due East, then another 4 miles East.
She then retraces her steps for 3 miles.
How far from her starting point is she?

Let x be the distance from the starting point.

$x = 5 + 4 - 3$ (*form the equation*)
$x = 6$ (*simplify constants*)

So the camel is 6 miles East from her starting point.

YOUR TURN!

Darius has 4 gold pieces.
A thief steals 2 of his gold pieces, but he then earns 5 more. How many gold pieces does Darius now have?

Let x equal the number of gold pieces he finishes with.

SHOW YOUR WORKINGS!

$x = 4 - 2 + 5$ (*Form the equation*)

$x = 2 + 5$ (*Simplify constants*)

$x = 7$

So Darius has 7 gold coins in the end.

Challenge 2: Adding to both sides

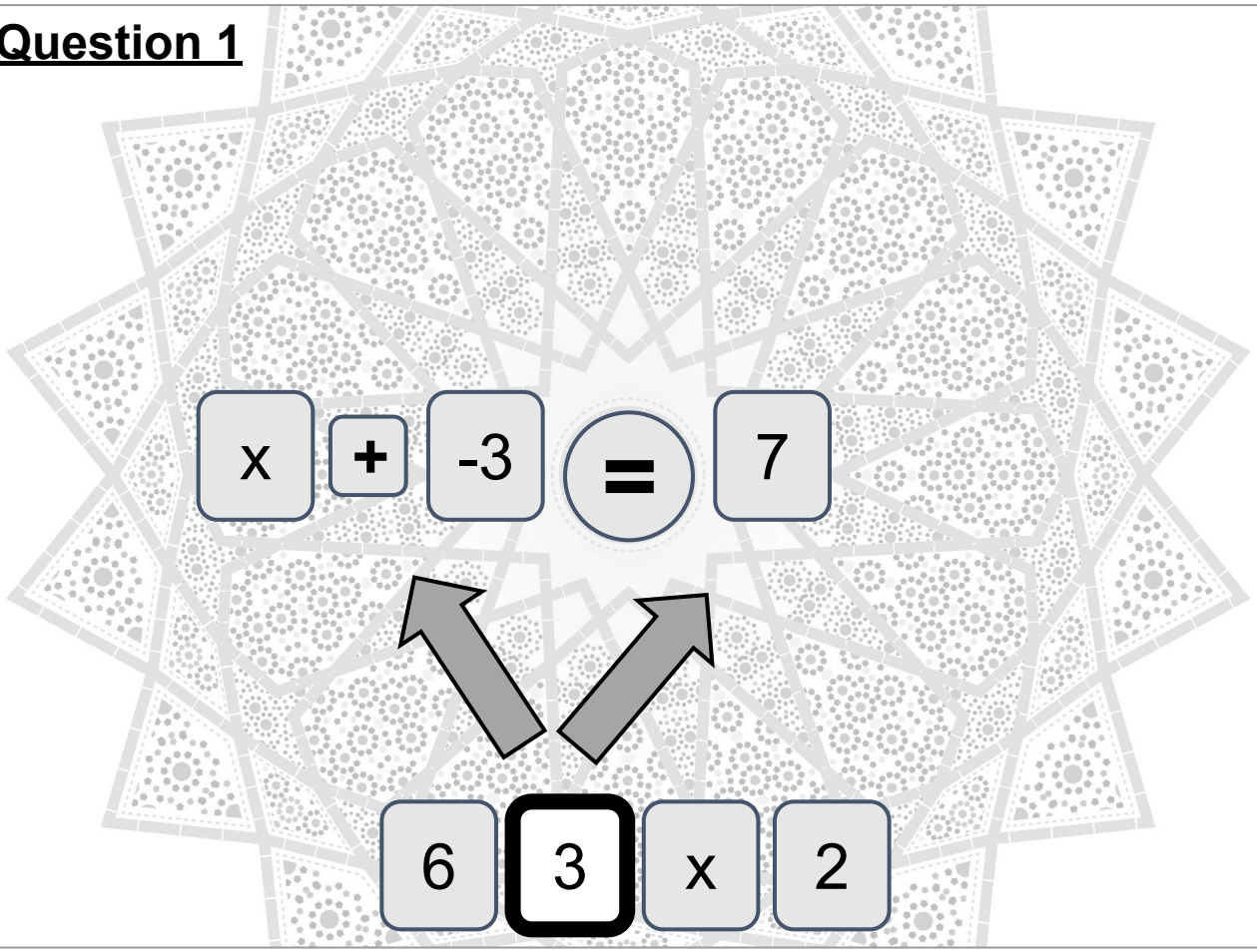
2.1 Adding to Both Sides

Lots of equations can be solved by
adding a constant to both sides of the equation.
Think of equations like scales.
Add the same amount to each side so that they balance.

Put a circle aroun t that you need to add to both sides of the equation to help solve it.

Question 1 is done for you.

Question 1



Question 2

-5 | 8 | 5 | 3

Question 3

6 | 0 | -1 | 1

Question 4

$x + 2 = 3 + 2$

3 | -x | -2 | -3

Question 5

$x + 1 + 8 = 6 + 5$

9 | -9 | -8 | 8

Challenge 2: Adding to both sides

2.2 Balancing Equations

It is important to show all your working when you solve an equation - even when you can '*see*' the answer!

Question 1

$$x + 4 = 9$$

Add to both sides..

$$x + 4 + \boxed{-4} = 9 + \boxed{-4}$$

$$x = \boxed{5}$$

HINT:
Use a number from this box

4	−4	9	−9
0	−5	5	1

Question 2

$$x - 2 + 3 = 5 + 4$$

Simplify constants

$$x + \boxed{1} = \boxed{9}$$

Add to both sides..

$$x + \boxed{1} + \boxed{-1} = \boxed{9} + \boxed{-1}$$

$$x + \boxed{0} = \boxed{8}$$

$$x = \boxed{8}$$

HINT:
Use a number from this box

1	−1	9	5
0	−5	8	10

Challenge 2.2 Balancing Equations cont

The variable doesn't have to be x...other letter symbols can be used!

Fill in the boxes to complete each solution.

Question 3

$$y - 6 = 9 - 4$$

$$y - 6 = \boxed{5}$$

$$y - \boxed{6} = \boxed{5}$$

Add to both sides

$$y = \boxed{11}$$

HINT:
Use a number from this box

6	−6	9	−9
11	−11	1	−1

Question 4 Now it's your turn….

$$y - 2 + 1 = 5 - 2$$

$$y - 1 = 3$$

$$y - 1 + 1 = 3 + 1$$

$$y + 0 = 4$$

$$y = 4$$

HINT:
Use a number from this box

1	−1	3	2
−2	−3	4	0

Question 5

$$4 + z - 2 = 1 + 3 - 5$$

$$z + 2 = -1$$

$$z + 2 + (-2) = -1 + (-2)$$

$$z + 0 = -3$$

$$z = -3$$

HINT:
Use a number from this box

1	−2	2	3
4	−1	0	−3

Challenge 2: Adding to both sides

2.3 Word Problems

EXAMPLE

A pomegranate and a jar of olives cost 12 bronze coins
A jar of olives costs 4 bronze coins.
How much does a pomegranate cost?

Let p represent the cost of one pomegranate.

$$p + 4 = 12 \quad \text{(}\textit{Form the equation}\text{)}$$
$$p + 4 + (-4) = 12 + (-4) \quad \text{(}\textit{Add to both sides to isolate } p\text{)}$$
$$p + 0 = 8 \quad \text{(}\textit{Simplify constants}\text{)}$$
$$p = 8 \quad \text{(}\textit{Remove zeros}\text{)}$$

So a pomegranate costs 8 bronze coins.

YOUR TURN!

Jila has some eggs.
She breaks 4 eggs and buys six more.
At the end she has 18 eggs.
How many did she start with?

Let x represent the number of eggs Jila starts with.

$x + (\ \boxed{-4}\ ) + \boxed{6} = 18$ (*Form the equation...*)

$x + 2 = 18$ (*...now solve it!*)

$x + 2 + (-2) = 18 + (-2)$

$x + 0 = 16$

$x = 16$

So Jila starts with 16 eggs.

Challenge 3: Coefficients, Like Terms, Multiplying Constants

3.1 True or False

Tick the statements which are always true.

Correct any statements which are false.

$1x = x$ ✔

$0x = 1$ ✘ $0x = 0$

$+3 + -3 = 0$ ✔

$(-4x) + (4x) = 0$ ✔

$-2 + -2 = 0$ ✘ $-2 + -2 = -2$

$5x + (-5x) = 0$ ✔

$4x + (-5x) = x$ ✘ $4x + (-5x) = -x$

$-4 + x + 4 = x$ ✔

3.2 Simplifying Like Terms

Simplify each expression in the left hand column of the table.

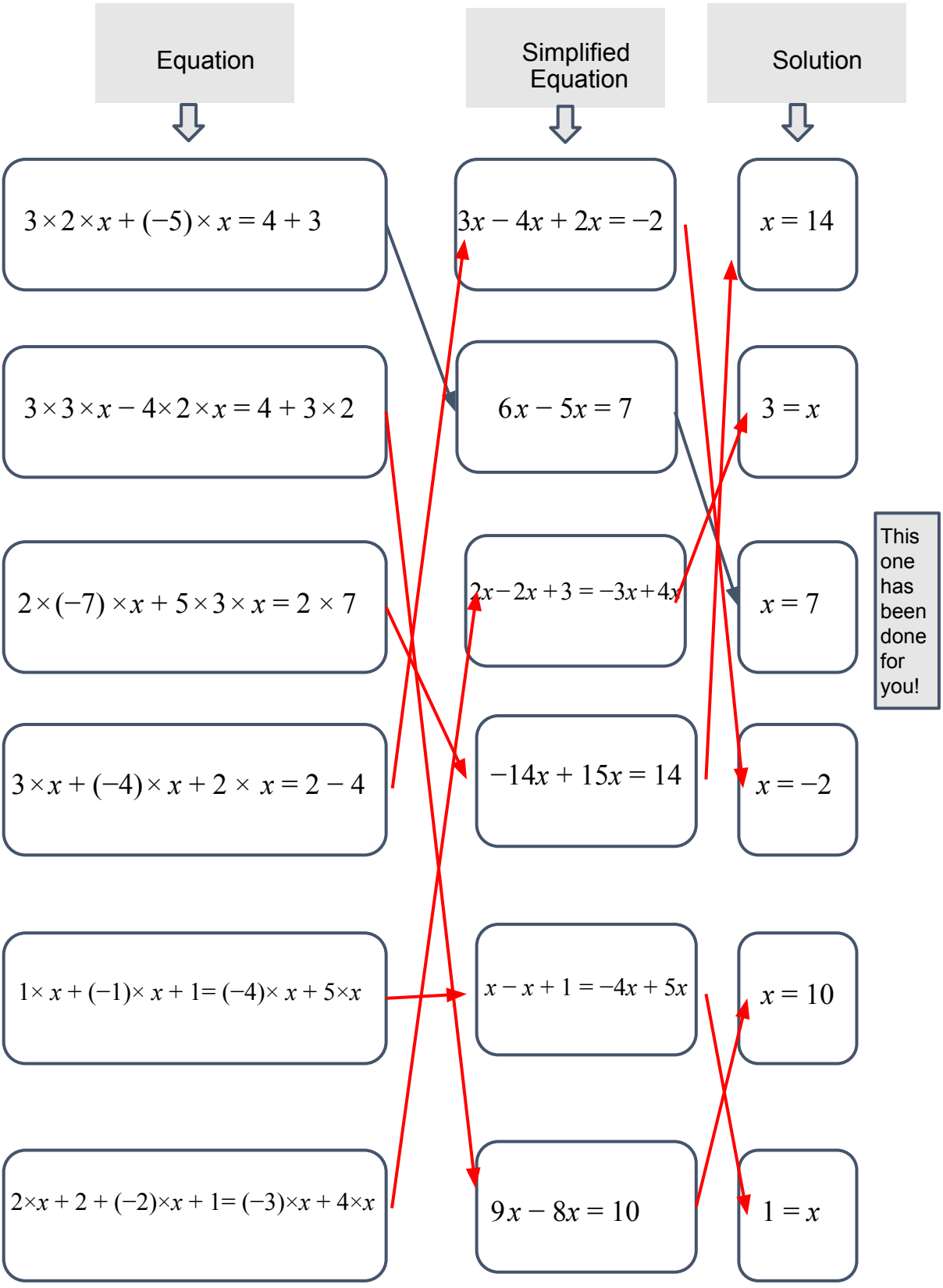
The first one is done for you!

Expression	*Simplified expression*
$5 \times (-3)$	-15
$4 + 2 \times 3$	10
$1x$	x
$-x + 2x$	x
$0x$	0
$-1x + x$	0
$x + x + x$	$3x$
$x + 2x - x$	$2x$
$-2x + 3 + 2x$	3
$-3 + 2x + 3$	$2x$
$-x + 2x + 3 - 3$	x
$6 - 3x - 3x - 2$	$4 - 6x$

Challenge 3: Coefficients, Like Terms, Multiplying Constants

3.3 Simplifying (Linking)

Alina has simplified each of these equations so she can solve them.
Match together the equations, simplified equations and solutions.



3.4 Simplifying and Showing Workings

Complete the missing working to show how you solve each equation.
You must always show each stage of your solution on a new line.

Question 1

$3x + 4x + 3 - 6x = 2x + 7 - 2x$

$x + 3 = 7$	*(simplify)*
$x + 3 + (-3) = 5 + (-3)$	*(isolate x)*
$x = 4$	*(simplify and solve!)*

This one is done for you...

Question 2

$5x + 4 - 4x = 3 \times 4$

$x + 4 = 12$	*(simplify)*
$x + 4 + (-4) = 12 + (-4)$	*(isolate x)*
$x = 8$	*(simplify and solve!)*

...now you do the rest!

Question 3

$6x - 2x - 2 - 3x = 2 \times 4 - 5$

$x - 2 = 3$	*(simplify)*
$x - 2 + 2 = 3 + 2$	*(isolate x)*
$x = 5$	*(simplify and solve!)*

Remember: The variable can be any letter!

Challenge 3.4 Simplifying and Showing Workings cont.

Question 4

$7x + 5 - 2 \times 3x = 3 - 2$

$x + 5 = 1$ *(simplify)*

$x + 5 + (-5) = 1 + (-5)$ *(isolate x)*

$\underline{x = -4}$ *(simplify and solve!)*

Explain what happens at each stage in the solution.

Question 5

$5 \times 2y - 4 - 3 \times 3y = 7 - 2 \times 3$

$y - 4 = 1$ *(simplify)*

$y - 4 + 4 = 1 + 4$ *(isolate y)*

$\underline{y = 5}$ *(simplify and solve!)*

You're on your own for these last two…

Question 6

$z + z + 3 \times 2 - 2z = 5z - 2 \times 2z + 3$

$6 = z + 3$ *(simplify)*

$6 + (-3) = z + 3 + (-3)$ *(isolate z)*

$\underline{3 = z}$ *(simplify and solve!)*

…don't forget to show all your working!

3.5 Word Problems

Example

Darius buys 2 bags of 4 melons and 3 bottles of oil.
On the way home, Darius sells 7 of his melons to Alina for the same price that he paid for them.
Altogether, Darius has spent 13 gold coins.

One bottle of oil costs 4 gold coins.
Find the cost of one melon.

Let m be the cost of one melon.

$2 \times 4m + 3 \times 4 - 7m = 13$ (*form the equation*)

$$8m + 12 - 7m = 13$$
$$m + 12 = 13$$
$$m + 12 + (-12) = 13 + (-12)$$
$$\underline{m = 1}$$

So a melon costs 1 gold coin.

Now you solve the equation!

Now it's your turn!

Alina buys 3 bags of 5 figs and 3 loaves of bread.
On the way home, Alina sells 14 of her figs to Jameela for the same price that she paid for them.
Altogether, Alina has spent 20 bronze coins.

One loaf of bread costs 6 bronze coins.
Find the cost of one fig.

SHOW YOUR WORKINGS.

Let f be the cost of one melon.

$$3 \times 5f + 6 \times 3 - 14f = 20$$
$$15f + 18 - 14f = 20$$
$$f + 18 = 20$$
$$f + 18 + (-18) = 20 + (-18)$$
$$\underline{f = 2}$$

So a fig costs 2 bronze coins.

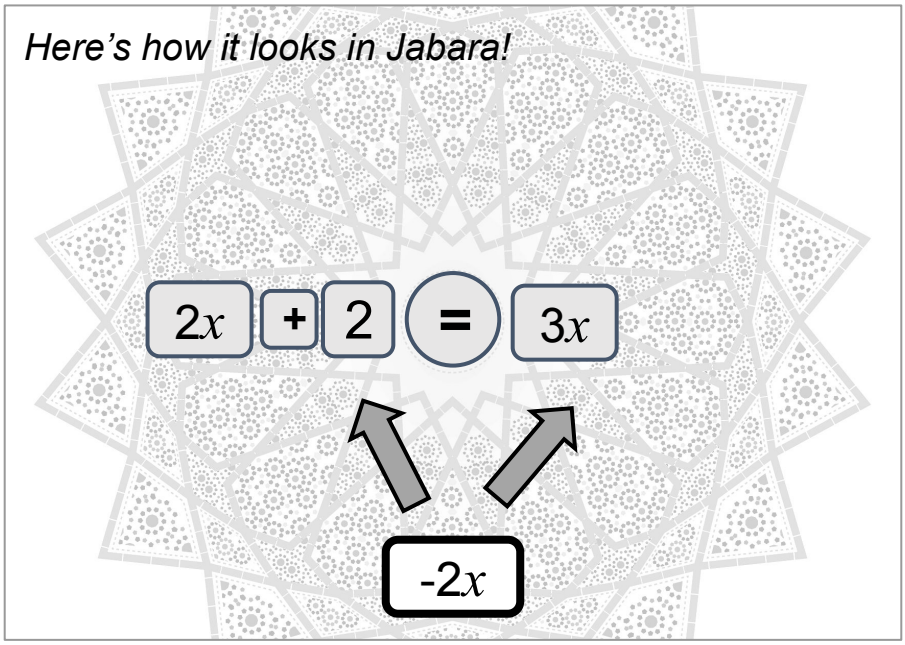
4.1 Adding variables to both sides

Sometimes x is on both sides of the equation!

You need to eliminate x from one side by adding negative x to BOTH sides to keep the equation balanced

Choose the right terms from the tray to get rid of the x on one side of each equation. You can use each term more than once!

Here's how it looks in Jabara!



Now it's your turn!

$3x + 3 = 2x + 1$

$3x + 3 +$ $= 2x + 1 +$

$(-2x)$ $=$ $(-2x)$ *(Simplify each side)*

x 1

(Complete the solution)

$x + 3 + (-3) = 1 + (-3)$

$x = -2$

Hint

$2x$ $-2x$

Challenge 4: Unknowns on both sides, adding variables, swapping sides

4.2 Spotting Mistakes

Alina's Algebra master has set her some equations to solve.
Here are Alina's solutions - but they are all wrong!
Find and correct all of Alina's mistakes.

$3x + 2x + 3 = 4x - 3$

$5x + 3 = 4x - 3$

$5x - 4x + 3 = -3$

$x + 3 = -3$

$x = -3 + 3$

$\underline{x = 6}$

$3x + 2x + 3 = 4x - 3$

$5x + 3 = 4x - 3$

$5x - 4x + 3 = -3$

$x + 3 = -3$

$x = -3 + (-3)$

$\underline{x = -6}$

$x - 2 + x = 3x - 5$

$-2 = 3x - 5$

$-2 + 5 = 3x - 5 + 5$

$3 = 3x$

$\underline{1 = x}$

$x - 2 + x = 3x - 5$

$2x - 2 = 3x - 5$

$-2 = 3x - 2x - 5$

$-2 + 5 = x - 5 + 5$

$\underline{3 = x}$

$-3x - x + 2 = 6 - 2x - x$

$-4x + 2 = 6 - 3x$

$x + 2 = 6$

$x = 6 - 2$

$x = 4$

$-3x - x + 2 = 6 - 2x - x$

$-4x + 2 = 6 - 3x$

$2 = 6 - 3x + 4x$

$2 = 6 + x$

$2 - 6 = x$

$\underline{-4 = x}$

Challenge 4: Unknowns on both sides, adding variables, swapping sides

4.3 Rearranging Equations

Use the maths hack to help you solve these equations.

$4x + 5x = 4x + 4x + 4$

$9x = 8x + 4$ (*Simplify*)

$9x - 8x = 4$ (*Put all the x's on one side*)

$\underline{x = 4}$ (*Simplify and solve!*)

$3x + 2x + x - 3 = 5x + 7$

$6x - 3 = 5x + 7$ (*Simplify*)

$6x - 5x - 3 = 7$ (*Put all the x's on one side...*)

$6x - 5x = 7 + 3$ (*...and the constants on the other*)

$\underline{x = 10}$ (*Simplify and solve!*)

$-6y + 3 = 3y + 2y - 10y + 4$

$-6y + 3 = -5y + 4$ (*Simplify*)

$3 = -5y + 4 + 6y$ (*Put all the y's on one side...*)

$3 - 4 = -5y + 6y$ (*...and the constants on the other*)

$\underline{-1 = y}$ (*Simplify and solve!*)

$7z - 5z - 2 = 5z - 2z - 6$

$2z - 2 = 3z - 6$ (*Simplify*)

$-2 = 3z - 6 - 2z$ (*Put all the z's on one side...*)

$-2 + 6 = 3z - 2z$ (*...and the constants on the other*)

$\underline{4 = z}$ (*Simplify and solve!*)

Try this one on your own!

4.4 Word Problems

Example

Darius and Alina go shopping at the market,
they both spend the same amount of money.
Darius buys 3 camels and 4 goats.
Alina buys 2 camels and 6 goats.

One goat costs 20 gold coins.
Find the cost of one camel.

Let c be the cost of one camel

$3c + 4 \times 20 = 2c + 6 \times 20$ (*form the equation*)

$3c + 80 = 2c + 120$ (*simplify the equation*)
$3c + 80 - 2c = 120$ (*move variables*)
$3c - 2c = 120 - 80$ (*move constants*)
$c = 40$ (*simplify and solve!*)

So a camel costs 40 gold coins.

Now solve the equation!

Now it's your turn!

Darius and Alina go shopping at the market,
they both spend the same amount of money.
Darius buys 4 cows and 7 sheep.
Alina buys 5 cows and 3 sheep.

One sheep costs 15 gold coins.
Find the cost of one cow.

Let c be the cost of one cow.

$4c + 7 \times 15 = 5c + 3 \times 15$ (*form the equation*)

$4c + 105 = 5c + 45$ (*simplify the equation*)
$105 = 5c + 45 - 4c$ (*move variables*)
$105 - 45 = 5c - 4c$ (*move constants*)
$60 = c$ (*simplify and solve!*)

So a cow costs 60 gold coins.

Challenge 5: Fractions, Dividing Both Sides

5.1 True or False

You have found some scrolls at the House of Wisdom, but some of them have been altered by Monge Khan.

Tick the statements on the scroll which are ***always true***.
Correct any statements which are false.

$\frac{15x}{15} = x$	✔	
$\frac{3x}{3} = 1$	✘	$\frac{3x}{3} = x$
$\frac{12x}{6} = 2x$	✔	
$\frac{10x}{10} = 0$	✘	$\frac{10x}{10} = x$
$\frac{3y}{6} = 2y$	✘	$\frac{3y}{6} = \frac{y}{2}$
$\frac{2 \times 4 \times 6x}{3 \times 4} = 3x$	✘	$\frac{2 \times 4 \times 6x}{3 \times 4} = 4x$

Challenge 5: Fractions, Dividing Both Sides

5.2 Code Breaker

The masters at the House of Wisdom were very secretive.
They used algebra to send coded messages.
Solve these equations in order to crack the code.

1. $3x = 9$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

A

2. $\frac{3 \times 4x}{12} - 2 = 4$

$\frac{12x}{12} - 2 = 4$

$x - 2 = 4$

$x = 6$ **L**

3. $\frac{6x}{2 \times 3} + 2x = 1 + 2 + 3$

$\frac{6x}{6} + 2x = 6$

$3x = 6$

$x = 2$ **G**

4. $6x = 6$

$\frac{6x}{6} = \frac{6}{6}$

$x = 1$

E

5. $\frac{20x}{2 \times 5} + 5 = 2 + 2 \times 2$

$\frac{20x}{10} + 5 = 6$

$2x = 1$

$x = \frac{1}{2}$ **B**

6. $18 = 10 - 4x$

$18 + 4x = 10$

$4x = 10 - 18$

$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$ **R**

3 = A	$\frac{1}{2}$ = B	−3 = C	1 = E	2 = G	0 = H
$\frac{1}{3}$ = K	6 = L	5 = N	−2 = R	−1 = S	4 = T

Challenge 5.2 Code Breaker cont.

7.

$2x - 3 = 3$

$2x = 3 + 3$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$ **A**

8.

$\frac{4x}{4} + 3 = 10 - 7$

$x + 3 = 3$

$x = 3 - 3$

$x = 0$ **H**

9.

$5x - 3 = 3x + 3$

$5x - 3x = 3 + 3$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$ **A**

10.

$x - 6 = x + x + x$

$x - 6 = 3x$

$-6 = 3x - x$

$\frac{-6}{2} = \frac{2x}{2}$

$-3 = x$ **C**

11.

$12x - 3 = 1$

$12x = 1 + 3$

$12x = 4$

$\frac{12x}{12} = \frac{4}{12}$

$x = \frac{1}{3}$ **K**

12.

$2x - 4 = 5x - 7$

$-4 + 7 = 5x - 2x$

$3 = 3x$

$\frac{3}{3} = \frac{3x}{3}$

$x = 1$ **E**

3 = A	$\frac{1}{2}$ = B
−3 = C	1 = E
2 = G	0 = H

13.

$x + 2x + 3x = 8x + 4$

$6x = 8x + 4$

$-4 = 8x - 6x$

$\frac{-4}{2} = \frac{2x}{2}$

$-2 = x$ **R**

$\frac{1}{3}$ = K	6 = L
5 = N	−2 = R
−1 = S	4 = T

Coded message: *Algebra Hacker!*

5.3 Word Problem

Can you solve a problem that is more than 1000 years old?

A man is hired to work in a vineyard
for 30 days for 10 gold coins.
He works 6 days.
How much should he receive?

Let x be the number of gold coins the man receives.

SHOW YOUR WORKINGS!

$$\frac{30x}{6} = 10$$

$$\frac{6 \times 5x}{6} = 10$$

$$5x = 10$$

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

So the man should receive 2 gold coins.

Challenge 6: Multiplying both sides, adding fractions with like denominators

6.1 Simplifying Equations (Linking)

Darius is solving some equations.

Here are his starting equations...

...and here is part of each solution.

Match together the equivalent equations.

The first one is done for you!

$\frac{5x}{5} = 2$	$x + 3 = 2$
$\frac{15x}{5} = 9$	$x = 2$
$\frac{x}{7} + 3 = \frac{2}{7} + 5$	$5 = x + 3$
$\frac{10 \times 2}{3} = x + 1$	$3x = 9$
$x + 3 = \frac{2}{5} + \frac{3}{5} + \frac{5}{5}$	$x + 21 = 2 + 35$
$5 = \frac{5x + 15}{5}$	$20 = 3x + 3$

Challenge 6: Multiplying both sides, adding fractions with like denominators

6.2 Solving Equations and Showing Workings

Mongke Khan has vandalized these scrolls.

Can you fill in the gaps?

Question 1

$$\frac{4x+8}{8} + \frac{x}{2} = 5$$

$$\frac{4x}{8} + \boxed{1} + \frac{x}{2} = 5$$ *(Split the fraction)*

$$\frac{x}{\boxed{2}} + \frac{x}{2} = \boxed{4}$$ *(Simplify and move the constant to the other side)*

$$\frac{x+x}{2} = 4$$

$$\frac{2x}{2} = 4$$

$$x = 4$$

Now finish off the solution!

Question 2

$$\frac{12x-9}{3} = x + 3$$

$$\frac{12x}{3} - \boxed{3} = x + 3$$ *(Split the fraction)*

$$4x - 3 = x + 3$$

$$4x - 3 - x = 3$$

$$3x = 3 + 3$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

Now finish off the solution!

Question 3

$$\frac{6-2x}{2} = \frac{5x+10}{5}$$

$$\boxed{3} - \boxed{x} = \boxed{x} + \boxed{2}$$

(Split the fractions)

$$3 = x + 2 + x$$
$$3 = 2x + 2$$
$$3 - 2 = 2x$$
$$\frac{1}{2} = \frac{2x}{2}$$
$$\frac{1}{2} = x$$

Now finish off the solution!

Question 4

Try this one on your own!

$$\frac{x}{60} + \frac{x}{20} = \frac{1}{3}$$

$$\frac{60x}{60} + \frac{60x}{20} = \frac{60}{3}$$

$$x + 3x = 20$$

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

6.3 Spotting Mistakes

Darius is training to become an algebra apprentice, but he keeps going wrong. Can you help him out? For each equation: find Darius's mistake, correct it and solve the equation.

$$3 = \frac{2x+4}{2}$$

$$3 = x + 4$$

$$3 - 4 = x$$

$$-1 = x$$

$$3 = \frac{2x+4}{2}$$

$$3 = x + 2$$

$$3 - 2 = x$$

$$1 = x$$

$$x = \frac{x}{2} - 4$$

$$x - \frac{x}{2} = 4$$

$$2x - x = 8$$

$$x = 8$$

$$x = \frac{x}{2} - 4$$

$$x - \frac{x}{2} = -4$$

$$2x - x = -8$$

$$x = -8$$

$$\frac{2x}{5} - \frac{x}{3} = 1$$

$$2x - \frac{x}{3} = 5$$

$$6x - x = 15$$

$$5x = 15$$

$$x = 3$$

$$\frac{2x}{5} - \frac{x}{3} = 1$$

$$2x - \frac{5x}{3} = 5$$

$$6x - 5x = 15$$

$$x = 15$$

6.4 Word Problems

Example

Alina travels by camel to the market at 4 leagues per hour to sell some silks.
On her way home, she travels at 5 leagues per hour.
Her total journey time is 9 hours.

How far away is the market from Alina's house?

Let d = the distance from Alina's house to the market

$\frac{d}{4} + \frac{d}{5} = 9$ (*form the equation*)

Hint: *distance = time / speed*

$d + \frac{4d}{5} = 36$

$5d + 4d = 180$

$9d = 180$

$d = 20$

So 20 leagues away.

Now you solve the equation!

Now it's your turn!

Darius goes for a run.
He leaves home and runs at 3 leagues per hour to the oasis.

On his way home, he runs at 2 leagues per hour.
His total journey time is 1 hour.

How far away is the oasis from Darius's house?

Let d = the distance from Darius's house to the oasis.

$\frac{d}{3} + \frac{d}{2} = 1$

$d + \frac{3d}{2} = 3$

$2d + 3d = 6$

$5d = 6$

$d = \frac{6}{5} = 1.2$

So 1.2 leagues away.

7.1 Expanding Brackets

Alina has expanded some expressions,
but her answers are all muddled up.
Can you sort them out?

Here are her starting expressions...

...and here are Alina's answers

Match together the equivalent expressions.

$6(2+4)$	$6x$
$2(2x+x)$	$6-x$
$3(4+x)$	36
$6(2-x)$	$3x-6$
$3(x-2)$	$12-6x$
$2(3+x)-3x$	$12+3x$

The first one is done for you!

7.2 Solving Equations and Showing Workings

Complete the working to solve each equation.

Question 1

$8 = 3(x + 1) + 2$

$8 = 3x + 3 + 2$ (*Multiply out the brackets*)

$8 = 3x + 5$ (*Simplify*)

$3 = 3x$ (*Move the constants onto one side*)

$1 = x$ (*And solve!*)

Question 2

$2(x - 4) + 3(x - 1) = 9$

$2x - 8 + 2x - 3 = 9$ (*Multiply out the brackets*)

$4x - 11 = 9$ (*Simplify*)

$4x = 20$ (*Move the constants onto one side*)

$x = 5$ (*And solve!*)

Question 3

$3(x + 1) - 2(4 - x) = 10$

$3x + 3 - 8 + 2x = 10$ (*Multiply out the brackets*)

$5x - 5 = 10$ (*Simplify*)

$5x = 15$ (*Move the constants onto one side*)

$x = 3$ (*And solve!*)

7.3 Word Problems

You'll need to use brackets to solve these word problems!

Darius, Alina and Jila share a bag of 185 gold coins.

Alina has 5 less coins than Darius.
Jila has twice as many coins Alina.

How many coins do they have each?

Let n = the number of coins that Darius has

Alina has $(n - 5)$ coins and Jila has $2(n - 5)$ coins

So $n + (n - 5) + 2(n - 5) = 185$

$n + n - 5 + 2n - 10 = 185$

$4n - 15 = 185$

$4n = 200$

$n = 50$

So Darius has 50 coins, Alina has 45 coins and Jila has 90 coins.

Form an equation first…

...and then solve it!

Darius is 6 years older than Alina.

6 years ago, Darius was twice Alina's age.

How old are Darius and Alina now?

Let d = Darius's age

Alina is $(d - 6)$ years old

6 years ago, Darius was $(d - 6)$ years old
and Alina wass $(d - 12)$ years old

So $(d - 6) = 2(d - 12)$

$d - 6 = 2d - 24$

$24 - 6 = 2d - d$

$18 = d$

So Darius is 18 years old and Alina is 12 years old.

Challenge 8: Fractions, Dividing Both Sides

8.1 Combining Fractions

Write each of the following as a single fraction.

Pair of fractions...	Single fraction...
$\frac{1}{2} + \frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{3} - \frac{1}{6}$	$\frac{1}{6}$
$\frac{x}{3} + \frac{x}{3}$	$\frac{2x}{3}$
$\frac{3x}{5} + \frac{x}{5}$	$\frac{4x}{5}$
$\frac{9x}{7} - \frac{3x}{7}$	$\frac{6x}{7}$
$\frac{2x}{10} + \frac{x}{5}$	$\frac{2x}{5}$
$\frac{6x}{12} - \frac{x}{4}$	$\frac{x}{4}$

Challenge 8: Fractions, Dividing Both Sides

8.2 True or False

You have found some scrolls at the House of Wisdom, but some of them have been altered by Monge Khan.

Tick the statements on the scroll which are ***always true***.
Correct any statements which are false.

$\frac{2x}{4} + \frac{x}{2} = x$	✔	
$\frac{5x + 10}{5} = x + 10$	✘	$\frac{5x + 10}{5} = x + 2$
$\frac{-4y - 2y}{2} = y$	✘	$\frac{-4y - 2y}{2} = -3y$
$\frac{15x}{10} - \frac{x}{2} = x$	✔	
$\frac{14y + 7}{7} = 1 + 2y$	✔	
$\frac{z}{2} - \frac{z}{3} = \frac{z}{5}$	✘	$\frac{z}{2} - \frac{z}{3} = \frac{z}{6}$

Challenge 8: Fractions, Dividing Both Sides

8.3 Solving Equations and Showing Workings

Complete the working to solve each equation.

Question 1

$$\frac{3x}{2} + \frac{4x}{8} = 10$$

$\frac{3x}{2} + \frac{x}{2} = 10$ (*Simplify the 2nd fraction*)

$\frac{4x}{2} = 10$ (*Add the fractions together*)

$2x = 10$ (*Simplify and solve!*)

$x = 5$

Question 2

$$\frac{1}{2} + \frac{8x + 4}{8} = 4$$

$\frac{1}{2} + \frac{8x}{8} + \frac{4}{8} = 4$ (*Split the 2nd fraction*)

$\frac{1}{2} + x + \frac{1}{2} = 4$ (*Simplify*)

$x = 4 - 1$ (*Move the constants onto one side*)

$x = 3$ (*And solve!*)

Question 3

$$\frac{5}{3} + \frac{6x + 5 - x}{15} = 1$$

$\frac{5}{3} + \frac{5x}{15} + \frac{1}{3} = 1$ (*Simplify and split the 2nd fraction*)

$\frac{x}{3} + 2 = 1$ (*Simplify*)

$x = -1$ (*Move the constants onto one side*)

$x = -3$ (*And solve!*)

8.4 Word Problems

Diophantus was known as the Father of Algebra.
Can you solve this ancient riddle to work out his age when he died?

Here lies Diophantus,' the wonder behold.
Through art algebraic, the stone tells how old:

His boyhood lasted one-sixth of his life;
his beard grew after one-twelfth more;
he married after one-seventh more;
and his son was born five years later;
the son lived to half his father's final age,
and Diophantus died four years after his son.

Let d = Diophantus's final age

$$d = \frac{d}{6} + \boxed{\frac{d}{12}} + \boxed{\frac{d}{7}} + \boxed{5} + \boxed{\frac{d}{2}} + 4$$

Complete the equation….

$$12d = \frac{12d}{6} + \frac{12d}{12} + \frac{12d}{7} + 12\times 5 + \frac{12d}{2} + 12\times 4$$

$$12d = 2d + d + \frac{12d}{7} + 60 + 6d + 48$$

….remove the fractions…

$$12d = 9d + \frac{12d}{7} + 108$$

$$3d - \frac{12d}{7} = 108$$

$$21d - 12d = 756$$

$$9d = 756$$

$$d = 84$$

Diophantus was 84 yrs old.

….then solve the equation.

Backup

Answers

- Exercise 1 – Variables and constants
  - Constants (circled) 1, 2, 17, 6, 8, 100
  - Variables (Square) x, p, y, t

- Exercise 2 – Removing zeros
  - $x = 2$ (already done), $x = 8$, $x = 1$, $x = 12$, $x = 3$, $x = 4$, $x = 7$

- Exercise 3 – Simplifying constants
  - $x = 6$ (already done), $x = 8$, $x = 4$, $x = 6$, $y = 17$, $y = 4$, $y = 9$

- Exercise 4 - Simplifying
  - $x = 7$ (already done), $x = 9$, $x = 6$, $x = 4$, $y = 3$, $y = 10$, $y = 6$

- Exercise 5 – Isolated variables
  - $x = 2 + 9$ (already done), $x = 5 + 7 + 3$, $y = 0 + 2 + 9$, $x = 7 - 4$

- Exercise 6 – Adding to both sides

(1) 4 (already done) (2) -6 (3) 4 (4) -5 (5) -3

- Exercise 7 – Solving equations

(1) $x = 5$ (already done) (2) $x = 8$ (3) $x = 6$

(4) $x = 4$ (5) $y = 14$ (6) $x = 8$ (7) $x = 5$ (8) $y = 6$

JABARA
Transferring Digital Skills to Paper

Game Challenge	example problem	Paper Transfer Exercise
L1 - L4	x = p + q	1-4
L5 - Lxx	x + 0 = p + q	2,4
Lxx - Lxx	x + p = q	5,6,7
Lxx - Lxx	-x = -p	7
Lxx - Lxx	x = q.p	
Lxx - Lxx	x = q.p / r	
Lxx - Lxx	px = q + r	
Lxx - Lxx	x + p/q = r/a + y/a	
Lxx - Lxx	p(x + r) = q	
Lxx - Lxx		
Lxx - Lxx		
Lxx - Lxx		
Lxx - Lxx		

Question 4

$x + 0 = 6$

$\square = 6$ (_____)

Write in what you are doing at each stage

Question 5

$0 + x = 12 - 8$

$x = 12 - 8$ (_____)

$x = \square$ *(simplify constants)*

Write in what you are doing at each stage

Question 6

$y + 0 = 4 + 3 + (-4)$

$y = 4 + 3 + (-4)$ (_____)

$y = \square$ *(simplify constants)*

The variable could be any letter! It is y here...

Question 7

$z + 0 = 4 + 3 + (-1)$

$\square = 4 + 3 + (-1)$ *(remove zeros)*

$z = \square$ *(simplify constants)*

...and it is z here!